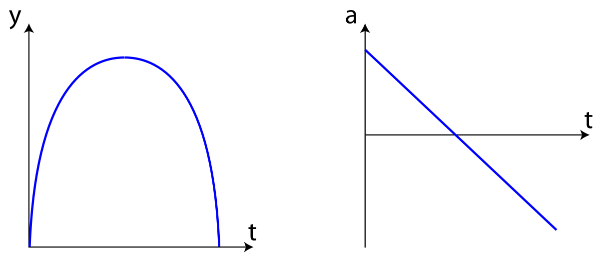
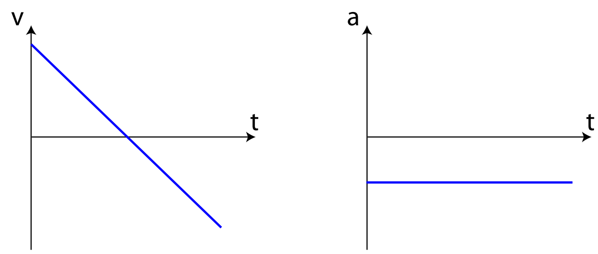
AP1 Kinematics

1. A ball is thrown vertically upward from the ground. Which pair of graphs best describes the motion of the ball as a function of time while it is in the air? Neglect air resistance.

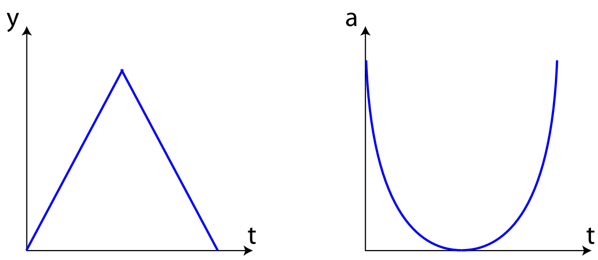
(A)



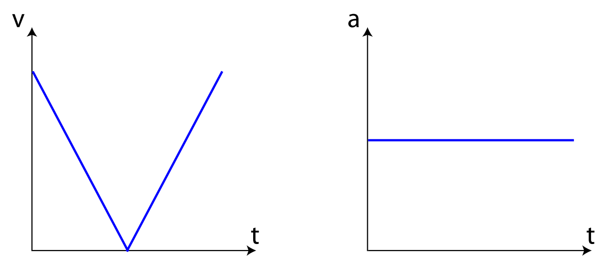
(C)



(B)



(D)



Answer: C

The velocity of the ball starts at a high initial value (assuming the upward direction is positive), then slows down at a constant rate until it reaches it highest point, at which point its velocity reaches an instantaneous value of zero. The ball then accelerates downward, increasing its speed at a constant rate until it returns to the same point in space and same speed it began its journey with. Throughout the entire time interval, the acceleration remains a constant negative value ($a=-g=-9.81\ m/s^2$ on the surface of the Earth).

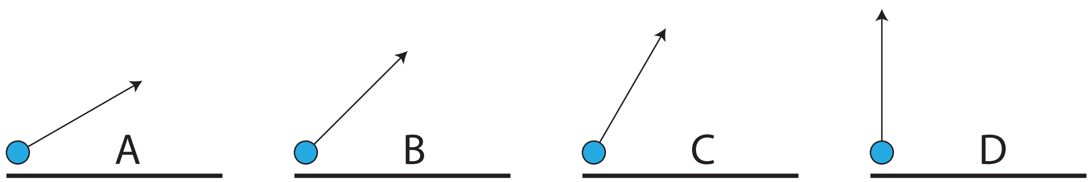
EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

SP: 1.1 The student can create representations and models of natural or man–made phenomena and systems in the domain.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations.

AP1 Kinematics

2. The following projectiles are launched on horizontal ground with the same initial speed. If two or three situations have the same answer, put the letters in the same blank space. Air resistance is negligible.



(a) Rank the situations from least to greatest in terms of time the projectile is in the air.
Least 1_____ 2_____ 3_____ 4_____ Greatest

Or, all of the projectiles are the same _________

(b) Rank the situations from least to greatest in terms of the horizontal distance the projectile travels while in the air.
Least 1_____ 2_____ 3_____ 4_____ Greatest

Or, all of the projectiles are the same _________

(c) Rank the situations from least to greatest in terms of the vertical distance the projectile travels while in the air.
Least 1_____ 2_____ 3_____ 4_____ Greatest

Or, all of the projectiles are the same _________

(d) Rank the situations from least to greatest in terms of the acceleration of the projectile while it is in the air.
Least 1_____ 2_____ 3_____ 4_____ Greatest

Or, all of the projectiles are the same _________

Answers:

(a) A, B, C, D -- the more vertical the vector the more time in the air since all initial velocities are the same.
(b) D, CA, B -- The 45° projectile has the greatest horizontal range.
(c) A, B, C, D -- D has the largest vertical component of initial velocity.
(d) All are the same acceleration.

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 2.3 The student can estimate numerically quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations.

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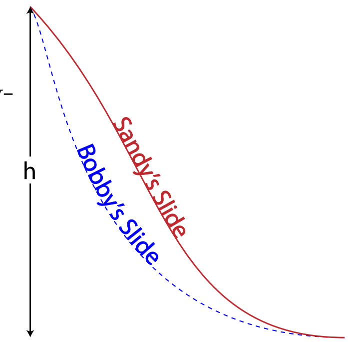


3. Two children on the playground, Bobby and Sandy, travel down slides of identical height h but different shapes as shown at right. The slides are frictionless. Assuming they start down the slides at the same time with zero initial velocity, which of the following statements is true?

(A) Bobby reaches the bottom first with the same average velocity as Sandy.
(B) Bobby reaches the bottom first with a larger average acceleration than Sandy.
(C) Bobby reaches the bottom first with the same average acceleration as Sandy.
(D) They reach the bottom at the same time with the same average acceleration.

Answer: B

Both children begin with gravitational potential energy mgh at the top of the slide, which is completely transferred to kinetic energy at the end of the slide. Bobby's potential energy is transferred more quickly, however, therefore he attains a higher average velocity and beats Sandy to the end of the slide. Average acceleration is the change in velocity divided by the time interval. Each child has the same change in velocity, but Bobby observes this change over a shorter period of time, resulting in a larger average acceleration.

EK: 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings. 5.B.4 The internal energy of a system includes the kinetic energy of the objects that make up the system and the potential energy of the configuration of the objects that make up the system.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.A.2.1 The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. 5.B.4.1 The student is able to describe and make predictions about the internal energy of systems.

4. You are asked to experimentally determine the acceleration of a skier traveling down a snow-covered hill of uniform slope as accurately as possible. Which combination of equipment and equation would be most useful in your endeavor?

	equipment	**equation**
(A)	tape measure, stopwatch	$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$
(B)	photo gates, stopwatch	$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$
(C)	radar gun, tape measure	$v_x = v_{x0} + a_x t$
(D)	photo gates, radar gun	$\overline{v}_x = \dfrac{v_{0x} + v_x}{2}$

Answer: A

Measure the time it takes the skier to go a set distance using the stopwatch, and use the tape measure to determine the distance. You may then use the equation $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ to solve for the acceleration, recognizing that the initial velocity and position of the skier are zero.

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.1 The student can justify the selection of a mathematical routine to solve problems. 4.2 The student can design a plan for collecting data to answer a particular scientific question.

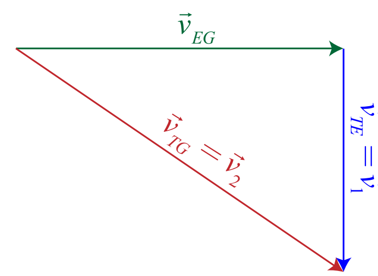
LO: 3.A.1.2 The student is able to design an experimental investigation of the motion of an object. 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system.

5. An eagle flies at constant velocity horizontally across the sky, carrying a turtle in its talons. The eagle releases the turtle while in flight. From the eagle's perspective, the turtle falls vertically with speed v_1. From an observer on the ground's perspective, at a particular instant the turtle falls at an angle with speed v_2. What is the speed of the eagle with respect to an observer on the ground?

(A) $v_1 + v_2$ (B) $v_1 - v_2$

(C) $\sqrt{v_1^2 - v_2^2}$ (D) $\sqrt{v_2^2 - v_1^2}$

Answer: D



Call the velocity of the turtle with respect to the eagle v_{TE}, also known as v_1.
Call the velocity of the turtle with respect to the ground v_{TG}, also known as v_2.
You are asked to find the velocity of the eagle with respect to the ground, v_{EG}.
Analyzing the right triangle, you can use the Pythagorean Theorem to solve for the magnitude of v_{EG}.

$$\vec{v}_{TG} = \vec{v}_{EG} + \vec{v}_{TE}$$

$$\vec{v}_{TG} = \vec{v}_{EG} + \vec{v}_{TE} \xrightarrow[\vec{v}_{TE}=\vec{v}_1]{\vec{v}_{TG}=\vec{v}_2} \vec{v}_2 = \vec{v}_{EG} + \vec{v}_1 \rightarrow$$

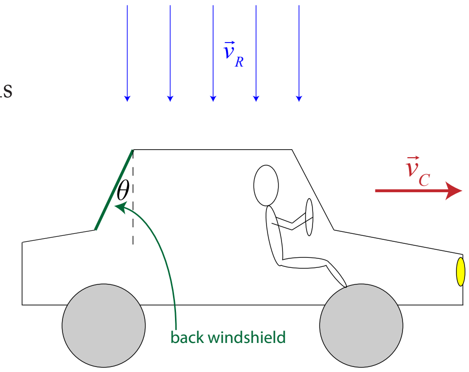
$$v_2^2 = v_{EG}^2 + v_1^2 \rightarrow v_{EG} = \sqrt{v_2^2 - v_1^2}$$

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system.

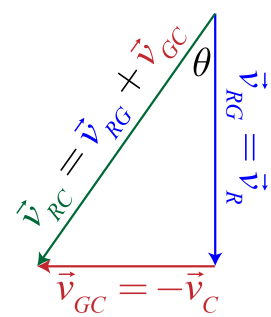
6. A car travels through a rainstorm at constant speed v_C as shown in the diagram at right. Rain is falling vertically at a constant speed v_R with respect to the ground. If the back windshield of the car, highlighted in the diagram, is set at an angle of θ with the vertical, what is the maximum speed the car can travel and still have rain hit the back windshield?



(A) $v_R \cos\theta$ (B) $v_R \tan\theta$

(C) $v_R \sin\theta$ (D) $v_R(1-\sin\theta)$

Answer: (B) $v_R \tan\theta$



In order for the rain to just hit the windshield, the angle of the velocity vector for the rain with respect to the car must match the angle of the back windshield. The velocity of the rain with respect to the car (v_{RC}) can be found as the vector sum of the velocity of the rain with respect to the ground (v_R) and the velocity of the ground with respect to the car ($-v_C$) as shown in the diagram at right. From this diagram, it is a straightforward application of trigonometry to find the speed of the car at which this condition occurs.

$$\tan\theta = \frac{v_{GC}}{v_{RG}} = \frac{v_C}{v_R} \rightarrow v_C = v_R \tan\theta$$

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system.

7. A cargo plane flies horizontally at a speed of 140 m/s at a height of 50 m above the ground. A supply package is dropped out of the bottom of the plane at time t=0. Two seconds later, a second package is dropped out of the bottom of the plane. Air resistance is negligible.

What happens to the separation between the packages as they fall through the air?

(A) The separation between packages decreases.
(B) The separation between packages increases.
(C) The separation between packages remains the same.
(D) Cannot answer without knowing the mass of the packages.

Answer: (B) The separation between the packages increases. The first package out of the plane has more time to accelerate while it is in the air, so it is moving at a faster speed than the second package dropped, therefore the separation between the packages increases while they fall.

EK: 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively.

LO: 4.A.2.1 The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time.

8. A cargo plane flies horizontally at a speed of 140 m/s at a height of 50 m above the ground. A supply package is dropped out of the bottom of the plane at time t=0. Two seconds later, a second package is dropped out of the bottom of the plane. Air resistance is negligible.

How far apart will the packages land on the ground?

(A) 70 m
(B) 140 m
(C) 280 m
(D) 420 m

Answer: (C) 280m. The packages will land on the ground at the same separation as the separation during their release, which is 140 m/s × 2s = 280m.

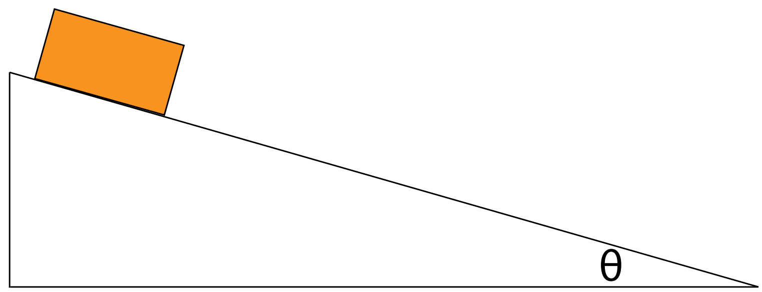
EK: 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 4.A.2.3 The student is able to create mathematical models and analyze graphical relationships for acceleration, velocity, and position of the center of mass of a system and use them to calculate properties of the motion of the center of mass of a system.

9. An object slides one meter down a frictionless ramp of constant slope as shown at right (not to scale). A student measures the time it takes for the object to travel various displacements using a stopwatch. Three consecutive trials are measured, and the data is recorded as shown below.

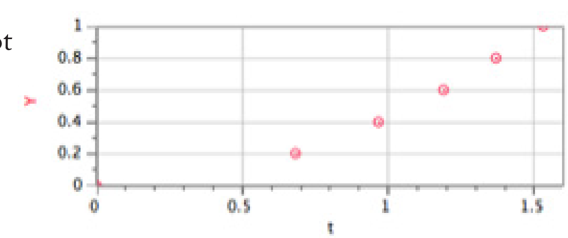
Displacement (m)	Avg. Time (s)
0	0
0.2	0.68
0.4	0.98
0.6	1.18
0.8	1.38
1	1.52



Determine the acceleration of the object.

Answer: 0.855 m/s^2

There are several methods of arriving at the answer, including, but not limited to, taking the slope of the v-t graph (calculating velocity for the various time intervals) or the use of the kinematic equations.



EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

SP: 1.1 The student can create representations and models of natural or man–made phenomena and systems in the domain. 2.3 The student can estimate numerically quantities that describe natural phenomena. 5.1 The student can analyze data to identify patterns or relationships.

LO: 3.A.1.3 The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations.

10. An object slides one meter down a frictionless ramp of constant slope as shown at right (not to scale). A student measures the time it takes for the object to travel various displacements using a stopwatch. Three consecutive trials are measured, and the data is recorded as shown below.

Displacement (m)	Avg. Time (s)
0	0
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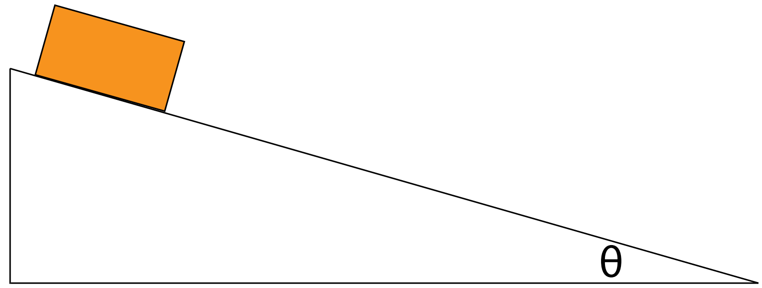
1	1.52



Determine the angle θ of the ramp.

Answer: 5°

After finding the acceleration of the object, recognize that the acceleration of the object is a=gsin(θ). Solving for theta, then, gives:

$$a = g\sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{a}{g}\right) = \sin^{-1}\left(\frac{0.855\,{}^{m}\!/_{s^2}}{9.81\,{}^{m}\!/_{s^2}}\right) = 5°$$

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. 3.B.2 Free-body diagrams are useful tools for visualizing forces being exerted on a single object and writing the equations that represent a physical situation.

SP: 1.1 The student can create representations and models of natural or man–made phenomena and systems in the domain. 2.3 The student can estimate numerically quantities that describe natural phenomena. 5.1 The student can analyze data to identify patterns or relationships.

LO: 3.A.1.3 The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. 3.B.2.1 The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively.

AP1 Kinematics

11. A pirate captain in her ship spies her first mate in a dinghy five kilometers away. The pirate captain sails her ship toward the dinghy at a rate of eight kilometers per hour. The first mate rows his dinghy toward the pirate ship at a rate of two kilometers per hour. When the captain initially spies the first mate at a distance of five kilometers, her parrot, Polly, begins flying back and forth between the two at a rate of 40 kilometers per hour. How far does Polly fly in total if she continues her back-and-forth journey until the pirate ship meets the dinghy?

Answer: 20 kilometers

The ship and dinghy approach each other at a combined 10 km per hour, therefore it takes them 0.5 hours to meet. During this entire time period, the parrot flies at 40 km/hr, therefore the total distance traveled by the parrot is 20 kilometers.

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. 4.A.2.1 The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time.

12. A train travels east toward Chicago at 80 km/hr. A man on the train runs from the front of the train toward the rear of the train at 10 km/hr. As he runs, he carries a plate of fruit with him. He notices a giant spider on the plate and throws the plate away from him (toward the rear of the train) at 20 km/hr. The startled spider jumps toward the man at 5 km/hr. The instant after the spider jumps toward the man, how fast is the spider approaching Chicago?

Answer: 55 km/hr

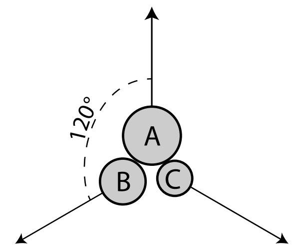
The spider's velocity with respect to the ground is the vector sum of the four given velocities, 80 km/hr east, -10 km/hr east, -20 km/hr east, and 5 km/hr east, for a resultant of 55 km/hr east.

EK: 3.A.1c An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration. A choice of reference frame determines the direction and the magnitude of each of these quantities. 4.A.2 The acceleration is equal to the rate of change of velocity with time, and velocity is equal to the rate of change of position with time.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. 4.A.2.1 The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time.

13. Three penguins are arranged in the center of a patch of frictionless ice floating across the ocean with a velocity of 2 m/s west. The mass of penguin A is 38 kg, the mass of penguin B is 30 kg, and the mass of penguin C is 23 kg. At time t=0, the penguins push off each other, each with a force of 20 newtons, such that they all slide away from the center of the floating ice patch at an angle of 120° from each other as shown in the diagram at right. Describe the motion of the center of mass of the three-penguin system at time t=3s.



Answer: The center of mass continues in its current state of motion, traveling west at 2 m/s.

Because there are no external forces on the system, the velocity of the center of mass of the system cannot be changed.

EK: 4.A.1 The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass. 4.A.3 Forces that systems exert on each other are due to interactions between objects in the systems. If the interacting objects are parts of the same system, there will be no change in the center-of-mass velocity of that system. 5.D.3a The velocity of the center of mass of the system cannot be changed by an interaction within the system. In an isolated system (a system with no external forces), the velocity of the center of mass does not change.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 4.A.1.1 The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semiquantitatively. 4.A.3.2 The student is able to use visual or mathematical representations of the forces between objects in a system to predict whether or not there will be a change in the center-of-mass velocity of that system. 5.D.3.1 The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force).

14. A fisherman in a small fishing boat at rest in a lake hooks a giant log floating in the lake 30 meters away. The fisherman reels the log in. During this process, the boat moves 12 meters in the direction of the log. If the mass of the boat and fisherman is 400 kg, what is the mass of the log? Assume frictionless.

Answer: m_{log} = 267 kg

Because there are no external forces on the system, and the center of mass is initially at rest, the center of mass must remain at rest. Set x=0 as the center of mass of the fisherman / log system:

$$x_{cm} = \frac{x_{fisherman} m_{fisherman} + x_{\log} m_{\log}}{m_{fisherman} + m_{\log}} = 0 \rightarrow x_{fisherman} m_{fisherman} + x_{\log} m_{\log} = 0 \rightarrow$$

$$m_{\log} = \frac{-x_{fisherman} m_{fisherman}}{x_{\log}} = \frac{(12m)(400kg)}{(18m)} = 267kg$$

EK: 4.A.1 The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass. 5.D.3 The velocity of the center of mass of the system cannot be changed by an interaction within the system.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 4.A.1.1 The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semiquantitatively. 5.D.3.1 The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force).

15. Two balls are launched off the edge a cliff of height h with an initial velocity v_0. The red ball is launched horizontally. The green ball is launched at an angle of θ above the horizontal. Neglect air resistance.

(a) Derive an expression for the time the red ball is in the air.

(b) Derive an expression for the horizontal distance traveled by the red ball while it is in the air.

(c) Derive an expression for the time the green ball is in the air.

(d) Derive an expression for the horizontal distance traveled by the green ball while while it is in the air.

(e) If the initial launch velocity v_0 of the balls is 100 m/s, the green ball is launched at an angle θ=30°, and the balls land 600 meters apart from each other, what is the height of the cliff? (*Note: calculator use strongly encouraged for this step*).

Note: This is a challenge problem that is algebra-intensive and outside the scope of what a student would see on an AP-1 exam.

Answers:

(a) $t_{red} = \sqrt{\dfrac{2h}{g}}$

(b) $\Delta x_{red} = v_0\sqrt{\dfrac{2h}{g}}$

(c) $t_{green} = \dfrac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}}{g}$

(d) $\Delta x_{green} = \left(v_0 \cos\theta\right)\left(\dfrac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}}{g}\right)$

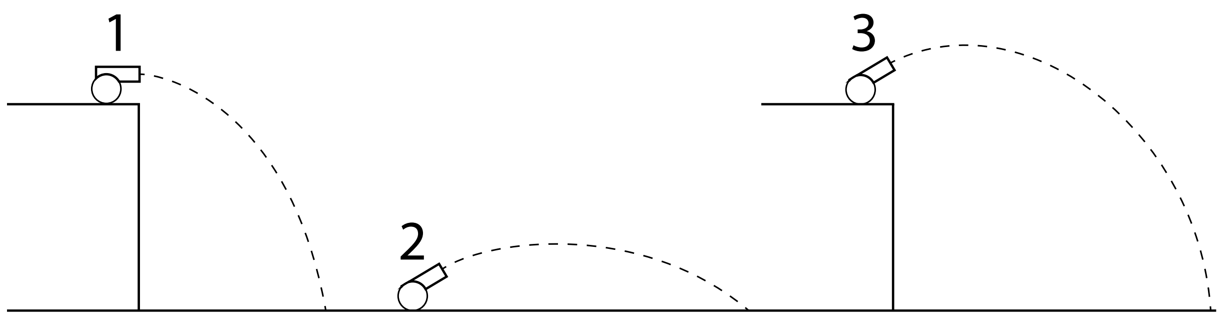
(e) H=81m

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

SP: 1.1 The student can create representations and models of natural or man–made phenomena and systems in the domain. 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 2.3 The student can estimate numerically quantities that describe natural phenomena.

LO: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations.

16. Using the following three scenarios, answer the questions that follow. All three cannons launch an identical mass with the same launch speed. The two angled projectiles are launched at the same angle above the horizontal. The two projectiles launched off a cliff are launched from the same height above the ground. Neglect air resistance. Illustrations are not drawn to scale.



(a) Rank the order of the resultant velocity at the projectile's maximum height from least to greatest. Explain your reasoning.

(b) Rank the order of the final resultant velocity just before the projectile hits the ground from least to greatest. Explain your reasoning.

Answers:

(a) 2 = 3 < 1

At maximum height, cannon 1 has a velocity of v. At their maximum heights, both cannons 2 and 3 have a velocity of vcos θ.

(b) 2 < 1 = 3

Cannon two has the lowest resultant velocity just before hitting the ground as its speed will be v. Cannons 1 and 3, however, both have speed v while they are at height h. They also then both travel the same vertical distance h to the ground, gaining the same amount of speed, resulting in the same final velocity. This can also be approached from an analytical perspective, solving for the final speeds of cannons 1 and 3 right before they hit the ground, obtaining $v = \sqrt{v_0^2 + 2gh}$ for both cannons.

EK: 3.A.1 An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.

SP: 1.1 The student can create representations and models of natural or man–made phenomena and systems in the domain. 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

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